Turbulent ship wakes: further evidence that the Earth is round

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When viewed from the stern, a ship's turbulent wake appears as a narrow strip of bubble-whitened water converging toward the horizon. The wake does not reach a sharp point on the horizon but has a finite angular width, indicating that the Earth is not flat, but rather round. A simple analysis of the geometry of the observations shows that the radius of the Earth can be estimated using only simple instruments and observations. © 2005 Optical Society of America

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1. Introduction

Viewed from aboard the ship, the turbulent stern wake appears as a long narrow whitish streak extending aft of the boat from the stern to the horizon (Fig. 1). Perspective gives the wake its characteristic triangular profile, just as mountain shadows appear triangular when viewed from the summit.[1] We wonder if ancient mariners recognized that their wakes were useful in determining the radius of the Earth. Its radius has been known to reasonable accuracy for over 2200 years. In about 500 B.C., Pythagoras[2] was the first to state that the Earth was round, though his arguments were based on the perfection of the sphere rather than on any observation. By around 350 B.C., Aristotle[3] had set forth three observations to prove that the Earth was round, but he did not estimate its size. Eratosthenes[4] (276–196 B.C.) was the first to measure it around 230 B.C. by using the altitude of the Sun measured simultaneously at two different latitudes. He found values of $4.78 \times 10^3 < R < 6.25 \times 10^3$ km, the latter being remarkably close to the true value of 6378.164 km.

Any observer would notice the blunted shape of the wake and finite angular width at the horizon. This might spark the idea that perhaps horizon was not infinitely far away. Any recognition of a finite distance to the horizon should also carry a recognition of curvature. After all, on a flat Earth, perspective would cause a long straight path to converge to a sharp point on the infinitely distant horizon. From a single observation, a careful observer might deduce that the Earth could be either round or cylindrical. Changing the ship's direction by 90 degrees and observing the same wake profile, however, would eliminate the cylindrical possibility. In this paper, we show that measurements of the turbulent wake's shape at the horizon could allow a sailor to deduce the Earth's spherical shape and its size.

2. Analysis

We have modeled Fig. 1 for two conditions, a flat Earth and a round Earth. In both cases, the wake width was set to $W = 30$ m, the ship's beam (Crystal Symphony). The height of the observer H was taken as 32 m based on measurements of the ship. The ship was in relatively calm waters in the Indian Ocean, and the atmospheric visibility was excellent. The horizon appeared sharp, and the turbulent wake was clearly evident all the way to it. The image scale of Fig. 1 was obtained from the focal length of the lens (75 mm). Based on measurements of Fig. 1, we found that the angular width of the wake at the horizon ω was $\sim$46 arcminutes, a value that is great enough to be easily discernible by eye. Any sailor could determine all of the necessary quantities ω, H, and W with relative ease.

Based on the value of ω, H, and W determined above, Fig. 2 shows the simulation for a flat Earth. Here, the ship wake is assumed to be infinitely long and converges to a sharp point on the horizon, an infinite distance away. This illustrates the appearance of the turbulent wake on a flat Earth in the absence of atmospheric extinction or refraction.

Figure 3 shows the geometry for the round Earth.

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Fig. 1. Ship's stern wake in the Indian Ocean on a clear day. The turbulent wake appears to converge toward the horizon but does not come to an absolutely sharp point. There is a noticeable angular width on the horizon. The wake is caused by bubbles from the ships propellers and turbulence from the ship. It has a nearly constant width and remains essentially fixed with respect to the water. (Apparent curvature of horizon is an optical effect due to the camera lens and is not representative of the Earth's curvature.)

The distance to the horizon D_T defined as the geometrical tangent point of the line-of-site with the Earth surface is

$$D_T = (2RH + H^2)^{1/2}, \qquad (1)$$

where R is the radius of the Earth and H is the observer's distance above sea level. For R = 6378.164 km and H = 32 m, D_T is ~20.2 km. The tangent point is depressed below the true horizon by an amount α_T,

$$\alpha_T = \sin^{-1}(D_T/(R + H)), \qquad (2)$$

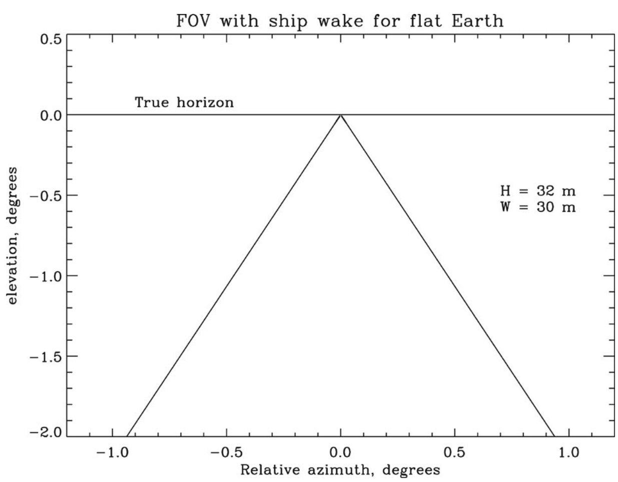

Fig. 2. Simulation of a ship wake on a flat Earth. The wake converges to a point in the true horizon. 90 degrees from the zenith (no refraction).

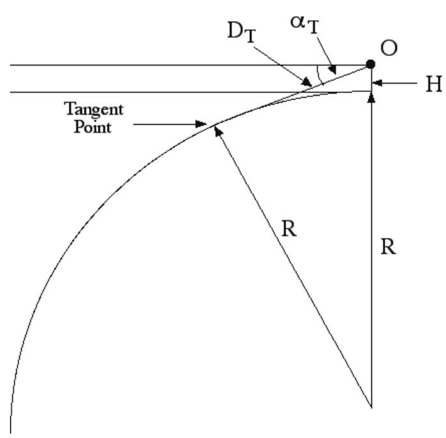

Fig. 3. Geometry of a ship wake on a round Earth. An observer O standing a distance H above a spherical (or cylindrical) Earth whose radius is R, will have a line of sight that is tangent to the Earth's surface at a distance D_T and an angle α_T below the geometrical horizon.

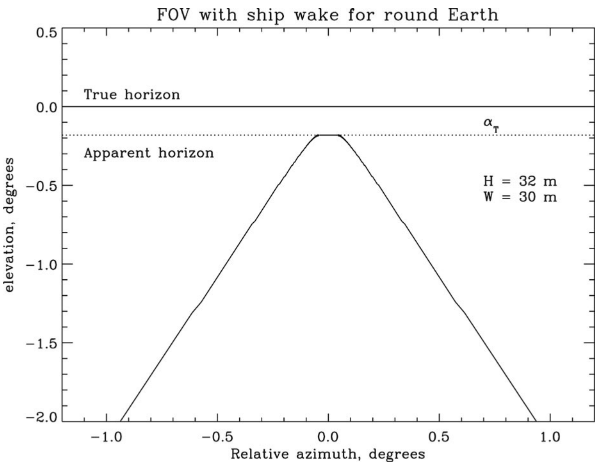

Fig. 4. Simulation of a ship wake on a round Earth. The wake converges but does not come to a point. It has a finite width of the apparent horizon, which is below the geometrical horizon by an amount α_T, the so-called "dip."

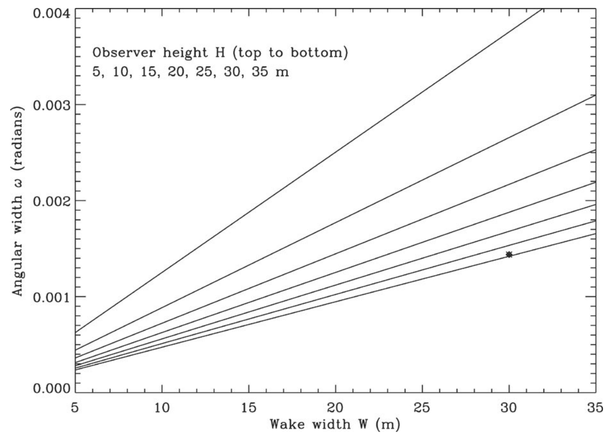

Fig. 5. $\omega(W)$ for various values of H. The dot marks the location of the calculations based on Fig. 1.

in this case 0.18 degrees. This angle is well known to celestial navigators[5] (e.g., Bowditch, 1995) and is called "dip." Equations (1) and (2) are almost trivial derivations, and they have been done by many authors.

Figure 4 shows the simulation of the wake on the real Earth. Near the horizon, it does not come to a sharp point but instead becomes slightly rounded and is flat topped at the apparent horizon. The angular width of the wake ω at the apparent horizon is simply

$$\omega = W/D_T, \tag{3}$$

where W is the wake's physical width. By substituting D_T from Eq. (1) into Eq. (3) and solving for R, we obtain

$$R = (W^2 - \omega^2 H^2)/(2H\omega^2), \tag{4}$$

where every quantity on the right-hand side is known or measurable. From $W = 30$ m, $H = 32$ m, and a rough measurement of $\omega = 4.6$ arcminutes, we find that $R = 7.85 \times 10^3$ km, tolerably close to the true value of 6.38×10^3 km given the simplicity of the analysis. We could also have substituted Eq. (1) into Eq. (2) and solved for R, but the dip could not be obtained from Fig. 1. Figure 5 shows $\omega(W)$ for various values of H for a reasonable range of expected observing conditions. Also shown is the position of the observations based on Fig. 1.

3. Discussion

The unrefracted distance to the horizon D_T is ~20 200 m, but the refracted value[6] is closer to 22 100 m. Using French's expression for distance analogous to Eq. (1) results in a cubic equation in R. The refracted solution was obtained from a web-based cubic equation solver (and checked with a second solver)[7] and found to be 6.55×10^3 km. We mention refraction here to show that accurate determinations are possible. But ancient sailors would not have had the training or tools to derive French's equation or solve a cubic equation. They might, however, have been able to draw Fig. 3 and solve the geometry for R, and this would have lead to a reasonably accurate determination of the radius of the Earth.

Could extinction play a role in the horizon's visibility by lowering the contrast of the sky–sea boundary to the point of hiding the actual horizon? Not in this case. According to Bohren and Fraser,[8] even in the clearest of air, the horizon ceases to be visible when the observer is higher than ~2000 m above the sea. In the case of Fig. 1, the air was extremely clear and the observer was only at an elevation of 32 m, making his judgement of where the horizon was, and therefore the angular width of the turbulent wake, reasonably secure.

Our assumption of a fixed width of the turbulent wake is not strictly true. The bubbles in the turbulent wake will gradually diffuse laterally, the amount being dependent on turbulent motions from the ship and its propulsion system. Observations and computations[9,10] show that the spreading rate is very slow, varying roughly as the one-fourth power of the distance behind the ship. Thus the width of the wake is its initially width, i.e., roughly the beam of the ship, to which a slowly growing component must be added. The effect of including wake broadening in the calculations above would be to increase the derived radius of the Earth.

Modern ships move much faster than old ships, and it is likely that modern wakes endure longer and with greater visual contrast than did those of older, slower ships. Being slower and not screw driven, it would have taken longer for an old sailing vessel to reach a point where their wake was on the horizon, and its wake may well have dissipated. On the other hand, older ships were smaller, and the height at which an observer could get to might only have been around 5–10 m, corresponding to a horizon distance of 8–11 km. At a typical speed of 2 m/s, it would have taken the old ships only 1.6 hours to reach a point where its wake was on the horizon. Old ships were not be as hydrodynamically sleek as modern ones and therefore may have produced more bubbles than a modern ship of comparable size (screws aside).

4. Conclusions

Based on the finite angular width of ship wakes at the horizon when viewed from the ship, we simulated the geometry on a flat and a round Earth. To an observer on the ship, the flat-Earth wake comes to a point at the horizon, but a round-Earth wake is flat on top because of the finite distance to the horizon. The apparent horizon is also depressed below the true horizon. Both the depression ("dip") and the width of the wake at the horizon can be used to deduce the radius of the Earth. We found that we could make reasonably accurate measurements of the Earth's radius using only the observed angular width of the wake on the horizon, knowledge of the ship's width, and observer's elevation.

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