Origin of the anthelion

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The formation of the anthelion is discussed. Previous theories by Bravais, Humphreys, and others are shown to be imcompatible with observation, or highly improbable. An explanation is set forth in which the anthelion is formed in common hexagonal columns with the *c* axis horizontal and with two side faces vertical. Light enters an upper oblique face, is reflected twice by the end and opposite vertical face, then emerges from the crystal through the lower oblique face on the same side of the crystal it entered. In agreement with the observations, this mechanism produces no anthelion when the solar elevation is greater than 46°.

INTRODUCTION

The anthelion is a diffuse colorless patch of light which occurs in cirrus clouds at the solar elevation opposite the sun.[1] The optical formation has long been known: two reflections by the vertically oriented right angle dihedral of an ice crystal. Such configurations reverse the direction of light in the horizontal plane regardless of the angle of incidence, but leaves the vertical component unchanged. Unfortunately ice is known to form in many crystal habits which display the necessary crystal faces and alignment possibilities. The purpose of this study is to establish which of these crystals is responsible for the anthelion.

PREVIOUS THEORIES

All of the earlier explanations of the anthelion correctly predict its most obvious properties: (1) location and (2) achromaticity. This second characteristic results from either external reflections or symmetric internal paths which cause the dispersion when the light enters the crystal to be "undone" when it leaves. However, all of the theories are questionable on the grounds of orientation mechanisms, crystal abundance or predicted intensity.

In his work of 1845 Bravais[2] suggested that the anthelion was formed by asymmetrically developed plates whose *c* axis was horizontal. The light immerges and emerges by the same vertical basal face [Fig. 1(a)]. This explanation is correct optically but is untenable from the standpoint of aerodynamics. Plates as thin as Bravais postulated are subject to strong lifting forces and would descend through the air with their *c* axis vertical.

Besson[3] has proposed a mechanism involving four-vaned composite crystals which have their *c* axes horizontal and two side faces vertical [Fig. 1(b)]. These zusammengesetzten krystallen are formed by four columns which are joined together at the vertices of their pyramidal terminations. Such crystals have been reported by Nakaya[4] and Bentley and Humphreys.[5] Magono and Lee[6] classify them as composite bullets C2a. It is reasonable to suppose that these crystals would fall as Fig. 1(b) suggests, but the author is unaware of a reference to their aerodynamic properties. Sunlight reflects off the adjacent faces on crystals 90° apart. The efficiency of this mechanism is greatest when the sun is on the horizon and decreases rapidly with increasing solar elevation. At all solar elevations the maximum reflection efficiency occurs when the angle of incidence in the horizontal plane is 45° and falls off rapidly on either side. When the angle is greater than 45°, much of the once-reflected light "falls through" the gap between the adjacent faces because they do not physically intersect. When the angle is less than 45°, only a fraction of the once-reflected light reaches the second face. Since the angle of incidence is generally not 45°, these considerations argue against Besson's mechanism. Moreover, the extreme scarcity of these beautiful crystals indicates that they would not contribute significantly to the anthelion.

Humphreys[7] makes use of another type of composite crystal, the so-called capped column, CPla. As usual, the crystal has its *c* axis horizontal and two faces vertical. The anthelion-producing mechanism is external reflection from the vertical side face and the basal cap. This cause can be questioned on two grounds. Special crystals are required and they do not commonly form. When they do form, they could only orient themselves as suggested if both ends were capped with plates of equal size, an unlikely event. Unequal caps would cause one end to drag more than the other and the crystal would descend at some angle other than horizontal. The other objection is more serious. Since the basal face is counterminous with the side face, the problem encountered with Besson's four-sided star is avoided. The crystal in Fig. 1(c) would not be seriously affected by solar elevation and would display anthelia equally at even the largest altitudes. This is not observed. Few if any anthelia have been observed above 45° and the occasional sightings are of poor quality and subject to doubt (Bravais). We conclude that Humphreys' mechanism is not responsible for the anthelion.

Visser[8] and Hastings[9] mention that the anthelion may be nothing more than an accumulation of light at the intersection of the anthelic arcs. This is doubtful because the anthelion is observed when no anthelic arcs are present (Bravais,[2] and Lacy *et al.*,[10] and vice versa Lilequist[11]). However, an anthelic arc could cause a false anthelion if it was to coincide with a small compact cirrus cloud possessing the proper crystals. The arc would appear locally much brighter at this position and it could be mistaken for the anthelion. This might also explain the occasional anthelion observed above or below the solar altitude.[12]

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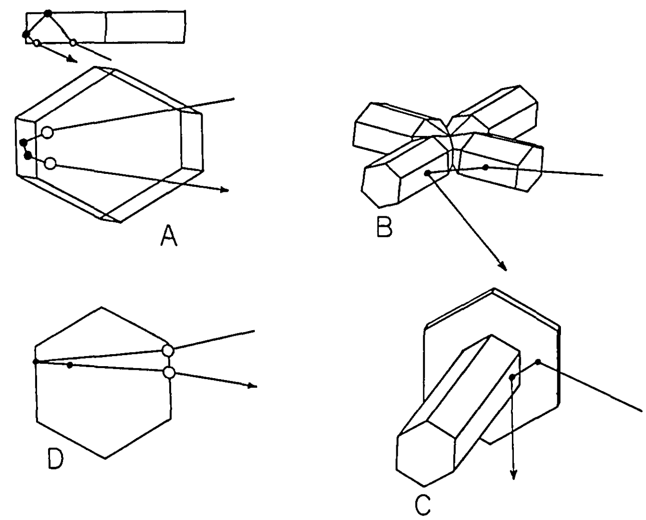

FIG. 1. Anthelion-producing mechanisms. The dots show reflections and the circles show transmissions. *A*, Bravais; *B*, Besson; *C*, Humphreys; *D*, partial contributor to the mechanism proposed in this paper.

PRESENT THEORY

There seems to be little doubt about the geometrical optics responsible for the anthelion. The question is in identifying the crystal. To do this we make use of the property of the anthelion mentioned earlier: It is common below 45° and virtually unknown above it. We propose the following explanation for the anthelion.

Simple columns with their c axis horizontal are oriented with two side faces vertical (Fig. 2 62E4 or 6E24). Light enters an upper oblique face and is reflected by the end face and the opposite vertical. These two reflections reverse the direction of the light in the horizontal plane. The ray then crosses the crystal and emerges through the lower oblique face on the same side of the crystal it entered. This path could also be attained in a single arm of the four-vaned crystal in Fig. 2(b).

The anthelion so produced is achromatic due to the symmetry of the light path relative to the refracting faces. It is easy to show that this mechanism produces an anthelion for all solar elevations below 45.66° and none above it.

Also, this explanation utilizes commonly occurring simple hexagonal prisms rather than rare polymorphic forms. Obviously, the same crystal [Fig. 1(d)] must also contribute to the anthelion at lower elevations ($\alpha < 21.3°$) when the light enters and exits through the same vertical side face.

In order to quantitatively test this model against the observations the relative cross section σ_0 as a function of α, the solar elevation, can be calculated and compared to the anthelion observations reported in the literature. The cross section is relative in the sense that it does not measure the true percentage of light which emerges from the crystal as an anthelion. It assumes that certain quantities are common to all phases of the calculation and consequently do not influence the shape of the curve. Figure 3 shows a cross section of the oriented crystal. It is drawn as a unit hexagon (each side is 1 unit long) and its image about the reflection face R is also shown to facilitate the computation. Figure 3(a) shows the geometry for $0 < \alpha < 19.08°$, where the maximum cross section occurs, and Fig. 3(b) shows it for $19.08 < \alpha < 45.66°$ (Fig. 2 62E4 or 6E24). The angle of incidence i and the angle of refraction r are related through Snell's law

$$\sin i = n \sin r \tag{1}$$

and i and r bear the following relations to α and β:

$$r + \beta = 60°, \tag{2}$$

$$i + \alpha = 60°. \tag{3}$$

The relative cross section σ_0 is

$$\sigma_0 = \sigma[\cos(60° - \alpha)]/[\cos(60° - \beta)], \tag{4}$$

where σ is the relative cross section inside the crystal and is defined as

$$\sigma = l \cos\beta, \tag{5}$$

where β is the apparent solar elevation within the crystal. σ is simply the diameter of the envelope of rays which successfully traverse the crystal in the prescribed manner. l is the effective length of the reflecting face R, which is diminished by shadowing, and is given by

$$l = 1 - 2(2\cos 30° \tan 30° - 2\cos 30° \tan\beta) = 4\cos 30° \tan\beta - 1. \tag{6}$$

Using (1), (2), and (3)

$$\beta = 60° - \sin^{-1}[\{\sin(60° - \alpha)\}/n] \tag{7}$$

Equations (4), (5), (6), and (7) can be used to calculate σ_0 for $0 < \alpha < 19.08°$ ($18.62° < \beta < 30°$).

$$\sigma_0 = (4\cos 30° \tan\beta - 1)\cos\beta \cos(60 - \alpha)/\cos(60 - \beta). \tag{8}$$

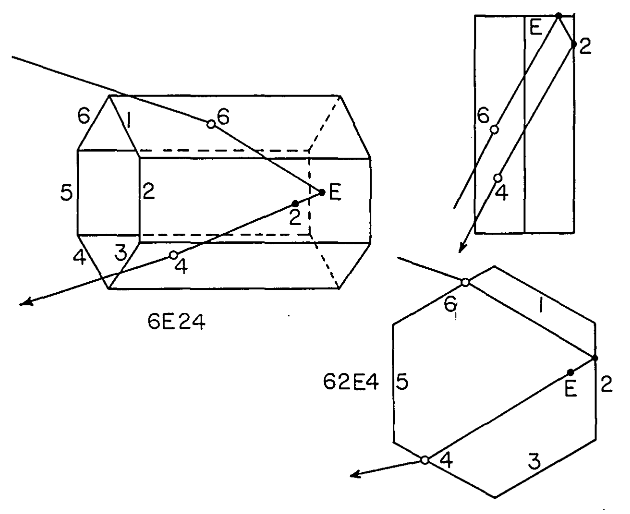

FIG. 2. Proposed mechanism for the anthelion. Light enters the crystal through an upper oblique face (6), and reflects off the end face (*E*) and the opposite vertical face (2) in either order. These two reflections reverse the direction of the ray in the horizontal plane. The light emerges from the crystal through the lower oblique face on the same side of the crystal it entered. This mechanism produces an anthelion up to 45.66° but is relatively inefficient below about 10°. The same crystal [Fig. 1(d)] produces the anthelion in a slightly different way for lower elevations.

For $19.08° < \alpha < 45.66°$ ($30° < \beta < 49.11°$) a similar analysis gives

$$\sigma_0 = (2 - 2\cos 30° \tan\beta)\cos\beta \times \cos(60° - \alpha)/\cos(60° - \beta), \quad (9)$$

where β is again obtained from Eq. (7). It is also necessary to calculate σ_0', the relative cross section for the mechanism shown if Fig. 1(d) (Fig. 2 5E25 or 52E5)

$$\sigma_0' = (1 - 4\cos 30° \tan\beta)\cos\alpha, \quad (10)$$

where

$$\beta = \sin^{-1}[(\sin\alpha)/n]. \quad (11)$$

At solar altitudes below 21.3° we would expect both mechanisms to act in concert, so the total cross section $\Sigma_0 = \sigma_0 + \sigma_0'$ must be calculated. Figure 5 shows σ_0 and Σ_0 as a function of α. Above $\alpha = 21.3°$, $\sigma_0' = 0$ and Σ_0 and σ_0 are identical. Above $\alpha = 30°$ one might ask if the light patch (Fig. 2) 1E23 or 12E3 would contribute to the anthelion. Such a path is optically impossible.

OBSERVATIONS

A search of the literature revealed 27 anthelion observations for which the solar elevation was given. Approximately twice this number were filed without the altitude. This regretable situation is probably due to the observer's lack of understanding of the importance of documenting all the attending circumstances. A histogram $N(\alpha)$ of the sightings was made by counting the number of anthelia which fell into bins 5° side.

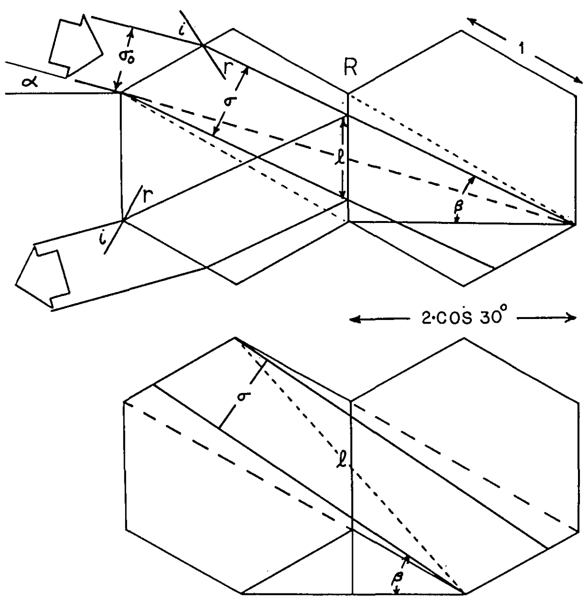

FIG. 3. Cross-section analysis of the anthelion-producing mechanism. The end view of the crystal is shown as a unit hexagon (face length = 1 unit) folded about R, the reflection face. σ_0 is the relative cross section of light which you can successfully traverse the crystal without being obstructed by the shadowed faces of the crystal. l is the length of the illuminated vertical face R which contributes to the anthelion. σ and β are the cross section and the solar elevation inside the crystal and are analogous to σ_0 and α. α is the true solar elevation.

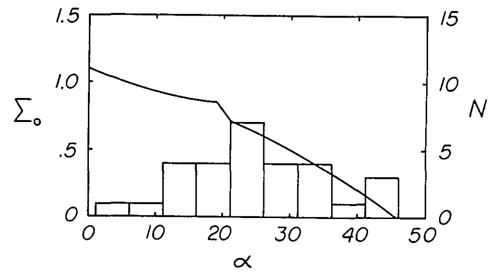

FIG. 4. Comparison of observation [$N(\alpha)$] and theory [$\Sigma_0(\alpha)$]. The thick curve shows $\Sigma_0(\alpha)$ the total relative cross section as a function of α. $\Sigma_0 = \sigma_0 + \sigma_0'$ below 19.08° and $\Sigma_0 = \sigma_0$ above it. The right-hand ordinate is $N(\alpha)$, the histogram of anthelion counts as a function of α in 5° bins. Both $N(\alpha)$ and $\Sigma_0(\alpha)$ go to zero near 46°.

$N(\alpha)$ is also plotted in Fig. 4. The vertical scales have been adjusted to achieve the best fit between $N(\alpha)$ and $\Sigma_0(\alpha)$. The essential quantity is the relative shapes of $\Sigma_0(\alpha)$ and $N(\alpha)$.

DISCUSSION

There is no *a priori* reason to believe that $N(\alpha)$ and $\Sigma_0(\alpha)$ can or should be compared to another. They are fundamentally different quantities, both in terms of units and physical origin. However, if one assumes that the frequency of observation is proportional to the brightness of the anthelion, then a comparison of $N(\alpha)$ to $\Sigma_0(\alpha)$ acquires some meaning.

The most important property of Fig. 4 is that both $N(\alpha)$ and $\Sigma_0(\alpha)$ go to zero at ~46°. This alone is compelling evidence that the mechanism proposed here is responsible for the anthelion. The general shapes of the curves are the same, especially above $\alpha \simeq 20°$. $N(\alpha)$ falls below $\Sigma_0(\alpha)$ below this elevation. This could be an observational selection effect. Most observers do not have an unobstructed view of the horizon. Trees, houses, mountains, etc. become increasingly obscured at lower elevations. Such an effect would hide anthelia and cause the observed number to decrease for small α. Ideally one would like to obtain $N(\alpha)$ from observations made at sea, but reports are not numerous enough. Besides, atmospheric attenuation increases exponentially as sec$z$ where $z = \pi/2 - \alpha$, the zenith distance. This would introduce a similar systematic decrease in observed anthelia.

The analysis here has assumed that the visibility of anthelia is proportional to the brightness, which in turn is a function of altitude. An improvement in the calculations might be achieved if the relative losses in the crystal are taken into account. Such a calculation is relevant only if Fresnel's equations are applicable, i.e., if the faces are smooth and the interior are without bubbles, striations, etc. Not enough is known about these crystals to warrant such a detailed analysis. However a rough calculation has been performed and the major effect seems to be to flatten the curve somewhat and broaden the peak. The salient feature remains: zero intensity at 45.66°.

The shape of $N(\alpha)$ is subject to additional systematic effects involving the population distribution and the occurance of cirrus clouds, both of which are a function of latitude. Since the world opoulation is generally equational (±50°), a greater number of anthelion sightings would be expected at middle and lower latitudes (high solar elevations). The latter would increase the number of halo observed at high latitudes where cirrus clouds are more common, thus augmenting $N(\alpha)$ at lower solar altitudes. At the present time, a quantitative treatment of these effects is not possible.

As Fig. 3 indicates, the light does not traverse the crystal in a plane perpendicular to the c axis because the c axes are randomly oriented relative to the solar azimuth in the horizontal plane. The calculations should use the *effective* index of refraction[14] which allows for the projected refraction as a function of the oblique angle of incidence. The effect on the calculation is minor, and the conclusions are unaltered.

Though the crystals proposed for the anthelion descend with their c axes horizontal and their side faces vertical, they will deviate slightly from strict alignment due to aerodynamic instabilities, collisions and turbulent buffeting. How much of a "tilt" is allowable? Obviously, the greater the departure, the larger and fainter the anthelion will be, and at sufficiently large tilts it will become indistinguishable from the background and vanish. It is easier to answer the question by turning it around and taking our cue from the observations. Anthelia are seldom larger than 4°–5° across. Allowing for a solar diameter of ½°, we deduce that the angular departures are of the order of 1°–2°.

CONCLUSION

The anthelion is probably formed in simple hexagonal columns which have their c axis horizontal and with two prisms faces vertical. The key evidence is the agreement between theory and observation that no anthelia are formed above 46°.

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The temperature dependence of collision-induced absorption in gaseous N_2

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Measurements of the absorption in compressed nitrogen gas in the spectral region 20–200 cm^{-1} have been made at temperatures of 200, 293, 320, and 353 K. Values of the electric quadrupole moment Q_{N_2} of the nitrogen molecules are determined from the results using two different analytical methods, at each of these temperatures, yielding an average value of $(1.3 \pm 0.2) \times 10^{-26}$ esu cm^2. The data are then applied in a study of the far-infrared transmission of the upper atmosphere at a mean temperature of 220 K, where it is shown that up to 8% absorption can occur in a vertical path above 12 km, while over a near-horizontal path from this level the absorption can be 40% or greater. Such results are of importance to atmospheric and astronomical measurements in the 100 cm^{-1} region.

I. INTRODUCTION

The purpose of this paper is to report measurements of the temperature variation of the far-infrared absorption in compressed N_2 gas. The particular aim of this work was to assess the importance of collision-induced absorption in N_2 in the Earth's upper atmosphere, where temperatures as low as about 220 K prevail.[1]

Previous to this work, several authors had observed the absorption in compressed N_2, but all at room temperature.[2–7] For problems in atmospheric physics, it is, therefore, important to know how the absorption varies with temperature, and the experiments reported here were designed to measure this variation.

The nitrogen molecule does not possess a permanent dipole moment, and so in the ground-vibrational (pure-rotational) state no absorption of electromagnetic radiation should take place. However, as a result of intermolecular collisions, transient dipole moments are induced in the molecules by interaction of their quadrupole electric fields, which permit absorption.[5] The pure rotation spectrum of N_2 lies in the 100 cm^{-1} (100 μm wavelength) region. Because the dipole moment induction involves two molecules the absorption is generally proportional to the square of density (pressure).

For reasons of concentration and relative absorption intensity, absorption by N_2 is of importance in the Earth's atmosphere only under conditions of low water-vapor concentration (the strong absorption by water vapor would otherwise

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