Step brightness changes of distant mountain ridges and their perception

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When successive ridges of distant mountains are seen, observers often report that, near the ridge where the brightness changes abruptly, the upper part of the nearer ridge appears darker than at its lower portions. Similarly, they report that the base of the more distant mountain seems brighter adjacent to the nearer ridge than on its upper portions. The explanation of this phenomenon, known as the step contrast effect, is a special case of Mach bands. It is usually attributed to a visual illusion involving lateral inhibition in the eye, which is most apparent in the vicinity of step brightness changes. Using analytic techniques and numerical integrations to simulate the airlight-induced brightness distributions of such scenes, we show that in many cases the perceived brightness distribution is qualitatively similar to the true brightness distribution and thus is not a visual illusion.
Key words: Mach bands, atmospheric optics, airlight, visual illusions, perception.

Introduction

Minnaert[1] remarks on the appearance of undulating moors and other sources of step brightness changes. He correctly notes that adjacent to the dark-bright step change at the top of the nearer moor, a visual illusion occurs. The nearer moor, being darker either by virtue of less intervening airlight than the more distant one or by the circumstances of the moors' surface or illumination, appears to have a dark band at its top bordering the more distant moor. Similarly, the more distant moor seems to have a bright band adjacent to the top of the nearer (darker) moor. Plates 21 and 22 are representative of such effects in distant mountains.

The visual illusion to which Minnaert ascribes the effect is discussed by Fiorentino and his collaborators[2,3] and Harms and Aulhorn[4] and is easily seen in Fig. 1. Figure 2 qualitatively shows the true and perceived brightness profiles. Loosely speaking, the perception is one of negative brightness gradients in the image, i.e., as the elevation angle increases, the brightness decreases except at the step change itself, above which the gradient is again negative. The presence of dark and bright bands in the perceived image is called the step contrast effect and is thought to be caused by lateral inhibition of adjacent photoreceptors in the human eye similar to the effect seen in the horseshoe crab, Limulus,[5–7] cats, and monkeys. Lateral inhibition is caused by inhibitory (modeled as negative) responses in the eye, which are shown schematically in Fig. 2 and which have been somewhat successfully modeled using differences and differentials of Gaussian functions.[8] The perceived bands are related to Mach bands[9] that occur where continuous brightness variations are present. As the width of the transition region narrows, the spatial gradient between light and dark increases, and ultimately becomes infinitely small. Here a step function occurs. In this sense the edge contrast effect is a special case of Mach bands.

From an image-processing standpoint, the eye is a medium-pass filter[10] that strongly suppresses lower frequencies and mildly suppresses higher frequencies. When the lowest frequency dc term of a stair-step function is filtered out, the result is similar to a sawtooth function, which would indeed cause bands to appear by virtue of the negative gradients in the perceived image.

For distant mountain ridges, airlight[11,12] is the source of the overall brightening of the scene with distance. The presumed photometric absence of bands in this stair-steplike function is usually explained in the following manner: for any realistic mountain, the ridge is more distant than any part of its face and the brightness would normally increase toward the ridge because there are more scatterers in the

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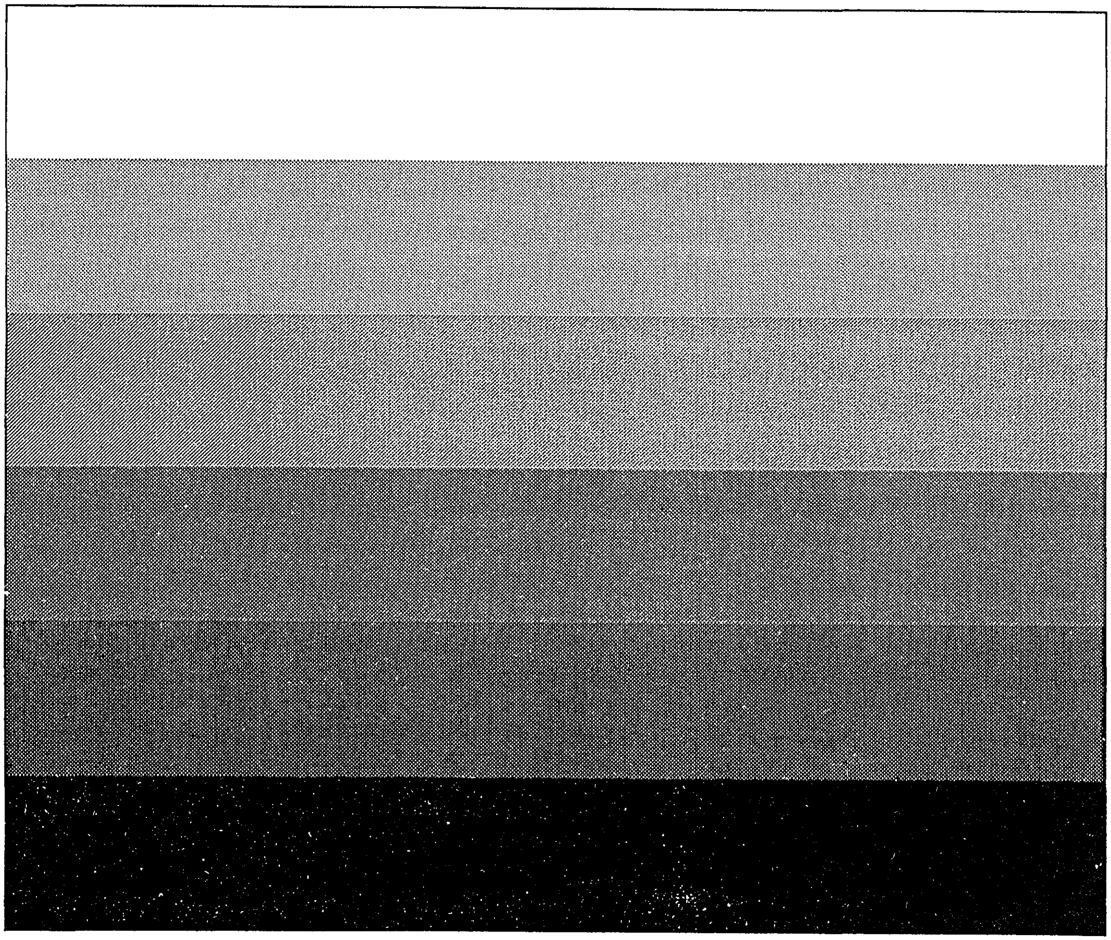

Fig. 1. Bands of uniform brightness. Note the bright and dark bands at the step brightness changes.

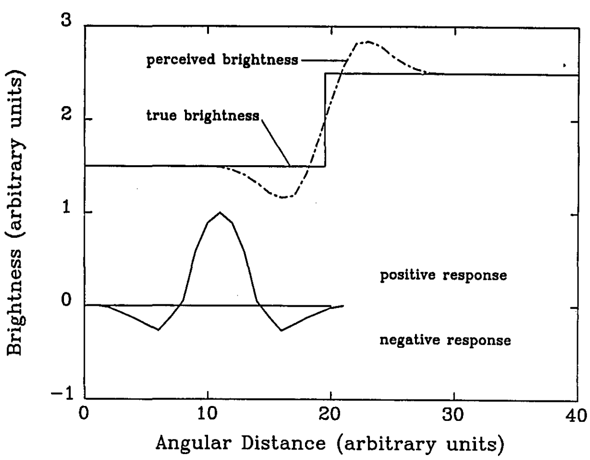

Fig. 2. True and perceived brightness distribution across a step in Fig. 1. The response function (lower curve) has negative values that produce lateral inhibition.

line of sight. Yet if the decrease in the number of scatterers with altitude makes up for the increased line-of-sight (LOS) distance, the brightness could decrease toward the ridge, i.e., the vertical brightness gradient would be negative.

Minnaert's observations refer primarily to nearby objects, or rather to those with a small interposed optical depth. In such circumstances the amount of intervening airlight would be negligible: moors and backlit buildings. Whether he would have been as firm in his recognition of the visual illusion in distant mountain ridges viewed through intervening airlight is not known.

Throughout the years, I have observed many such scenes and recognized the visual illusion. Yet there were times when it was so evident that I thought that it might not always be an illusion, i.e., that sometimes the true brightness distribution was in some way similar to the perceived brightness distribution. Of course, any visual assessment of the photometric properties of such mountain ridges is tricky: even if the photometric profile mimics that of the perceived illusion, the illusion will exaggerate the effect and thereby enhance its visibility. But in discussing my perception of bands in mountain ridges, a number of scientists have gently chastised me and opined that all I was seeing were visual illusions, namely, Mach bands. It was this apparently common identification of visual illusions in distant mountains in association with airlight that led to this work.

Theory

Figure 3 shows a schematic of the geometry used to analyze the problem. An observer at height *ho* views a series of mountain ridges at an elevation angle α below

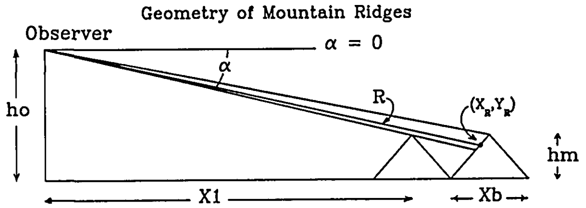

Fig. 3. Schematic diagram of distant mountain ridges used to model the viewing circumstances.

the horizon. The mountains are assumed to be perfectly black (or at least intrinsically uniform in brightness) and are located at a distance $X1$ and of height hm. The ridges are separated by a distance Xb. A scattering medium is present whose scattering number density $n(z)$ decreases exponentially with height z and is characterized by the vertical scale height Ho, i.e., $n(z) = n_0 \exp(-z/Ho)$. Using two straight lines, $z = ho + x \tan\alpha$ and $z = A + x \tan\beta$, which define the LOS and the mountain face with slope $\tan\beta$, respectively, we can solve for the position (x_R, y_R) where the LOS meets the mountain. Note that α is negative below the horizon and A is a geometric constant that changes from mountain to mountain. We have assumed that the angular scattering properties of the scatterers are constant over the small range of scattering angles (sun–scatterer–observer) defined by α. Physically this means that either the scatterers are large or that the sun is high in the sky where forward scattering, which usually changes rapidly with scattering angle, is unimportant. Apart from this caveat, the discussion is valid for an arbitrary scatterer.

How does the apparent brightness of the mountain (or more properly the airlight along the LOS to the mountain) change with elevation angle α? By inspection we might expect two regimes: If Ho is large, as is the case when air is the source of airlight, there would be a relatively constant number density of scatterers along the various LOS's. A small Ho, on the other hand, which might, for example, correspond to fog-filled valleys, would lead to a significant change in the number density of scatterers along the LOS as a function of α.

Because the top and bottom of one mountain face can show both dark and bright bands, respectively, it is sufficient to consider the vertical brightness $I(\alpha)$ of the mountain face between 0 and R:

$$I = K \int_0^R n(z)\mathrm{d}r = K \int_0^R n_0 \exp(-z/Ho)\mathrm{d}r, \tag{1}$$

where n_0 is the particle density at zero altitude and K is a constant that, in the discussion that follows, is omitted because it plays no part in the analysis. We have implicitly assumed that the solar irradiance is constant along the LOS and with elevation and that the type of scatterers does not change with either height or position along the LOS. For an optically thin scattering medium, we believe these assumptions are correct. Substituting $z = ho + r \sin\alpha$ and $\mathrm{d}r = dz/\sin\alpha$ and integrating we have

$$I(\alpha) = -n_0 Ho \exp(-ho/Ho)\{\exp[-(R/Ho)\sin\alpha] - 1\}/\sin\alpha. \tag{2}$$

Note the explicit dependence of I on α as well as the implicit dependence of R on α, which we make use of later.

If Ho is large, i.e., $Ho \gg R \sin\alpha$ and $Ho \gg ho$, the argument of the exponential in Eq. (2) is small and the exponential can be expanded as a power series of $1 - (R/Ho)\sin\alpha + \ldots$, leading to

$$I(\alpha) = -n_0 Ho \exp(-ho/Ho)[-(R/Ho)\sin\alpha]/\sin\alpha, \tag{3}$$

or

$$I(\alpha) = n_0 \exp(-ho/Ho)R. \tag{4}$$

The gradient $\mathrm{d}I/\mathrm{d}\alpha$ is equal to $n_0 \exp(-ho/Ho)\mathrm{d}R/\mathrm{d}\alpha$. This corresponds to the case where, for example, the airlight is the result of air molecules and there is no appreciable change of air density along the LOS. For any reasonable mountain, $\mathrm{d}R/\mathrm{d}\alpha > 0$ because, as the LOS approaches the horizon, the distance R to the mountain increases and thus $\mathrm{d}I/\mathrm{d}\alpha$ is always greater than zero, i.e., $\mathrm{d}I/\mathrm{d}\alpha > 0$. The transition to $\mathrm{d}I/\mathrm{d}\alpha < 0$ occurs when the LOS is perpendicular to the mountain face, i.e., when $\tan\alpha = -\cot\beta$, an unrealistic situation because the mountain face would have to be nearly vertical, leading to a $\mathrm{d}I/\mathrm{d}a$ that is zero because $\mathrm{d}R/d\alpha$ would be zero.

Because the above gradient $\mathrm{d}I/\mathrm{d}\alpha$ is always positive for a large Ho, there is nothing to suggest that the brightness distribution would mimic the perception of bands. Thus, for large Ho, any perceived bands would truly be visual illusions.

If Ho is small and $Ho \ll R \sin\alpha$, both the argument of the exponential and the exponential are large and

$$\exp[-(R/Ho)\sin\alpha] - 1 \sim \exp[-(R/Ho)\sin\alpha], \tag{5}$$

and Eq. (2) becomes

$$I = -n_0 Ho \exp(-ho/Ho)\exp(-R \sin\alpha/Ho)/\sin\alpha. \tag{6}$$

To find if $\mathrm{d}I/\mathrm{d}\alpha$ can be negative, we calculate $\mathrm{d}I/\mathrm{d}\alpha$ using the explicit expression for $R(\alpha)$,

$$R(\alpha) = [(ho - A)/(\tan\beta - \tan\alpha)]\sec^2\alpha, \tag{7}$$

and set $\mathrm{d}I/\mathrm{d}\alpha$ equal to zero. By noting that

$$\mathrm{d}R/\mathrm{d}\alpha = R[1 + \sec^2\alpha/(\tan\beta - \tan\alpha)], \tag{8}$$

we can then solve for Ho:

$$Ho = R \cos\alpha \tan\alpha + RC \sin\alpha \tan\alpha, \tag{9}$$

where

$$C = \tan\alpha + [\sec^2\alpha/(\tan\beta - \tan\alpha) + \cos\alpha]. \tag{10}$$

For small values of α, $\tan\alpha = \sin\alpha$, $\cos\alpha = 1$, and Eq. (9) reduces to

$$Ho = R \tan\alpha(1 + C \sin\alpha). \tag{11}$$

By further virtue of α's smallness, $\tan\alpha \ll 1$, $\sec^2\alpha = 1$, and $\cos\alpha = 1$, and, providing that $\beta \gg \alpha$, we can see that $C = 1$ and thus

$$Ho = R \tan\alpha = R \sin\alpha \tag{12}$$

when α is small. But from Fig. 5 we see that $R \sin \alpha$ is the vertical distance below the horizon that the LOS intersects the mountain face. Thus if $Ho > R \sin \alpha$, then $dI/d\alpha > 0$, and any perceived bands must be visual illusions. If $Ho < R \sin \alpha$, the gradient $dI/d\alpha < 0$ and the true brightness profile mimics the perceived one.

If the observer is high compared to the mountains, i.e., $ho \gg hm$, then, for $dI/d\alpha$ to be less than zero,

$$Ho = ho. \tag{13}$$

When the observer's altitude is only slightly higher than the mountains, the value of Ho depends on how low the LOS intersects the mountain face, which in turn depends on the mountain's slope $\tan \beta$. To first order then,

$$Ho = hm. \tag{14}$$

Owing to the relative complexity of the analytic approach as well as the number of approximations necessary to arrive at a simple solution, we choose to check the results numerically by integrating Eq. (1). The advantage of such computations is that $dI/d\alpha$ can be calculated using no assumptions and the results can be displayed graphically.

Numerical Analysis

The integration of Eq. (1) was performed numerically for the following conditions: $ho = 1000$ m, $X1 = 5000$, $hm = 500$ m, $Xb = 2000$ m. The scale height Ho was varied between 100 and 10,000 m for each set of computations (for clear air $Ho = 8400$ m, while for particles Ho is typically between 1000 and 2000 m). The values of all the parameters were chosen so as to be physically reasonable although in no way meant to include most possible situations while at the same time avoiding idiosyncratic or special-case values. Throughout the calculations we assumed that the LOS through the scatterers was optically thin, i.e., $\ll 1.0$. Tests with horizontal optical depths ranging between 0.1 and 10 clearly revealed the existence of the bands although their contrast was different from the optically thin case. For clarity in presentation, we calculated $I(\alpha)$ in the optically thin case for five successively more distant mountain ranges. The range of α between the tops of adjacent ridges was divided into 100 parts as was the distance R. I was computed using the sum

$$I = \sum_i n(r_i)\Delta r_i, \tag{15}$$

which is the finite difference form of Eq. (1).

Figure 4 shows the LOS brightness profiles for each scale height obtained by simple numerical integration, and Fig. 5 shows their vertical gradients $dI/d\alpha$ also computed numerically using a three-point Lagrangian interpolator. The upper curve in each figure closely approximates the limiting case where the scattering medium is uniformly distributed in height, i.e., Ho is large. Because $I(\alpha)$ is proportional to R, dI/dR and $dI/d\alpha$ are both greater than zero. The gradient $dI/d\alpha$ is

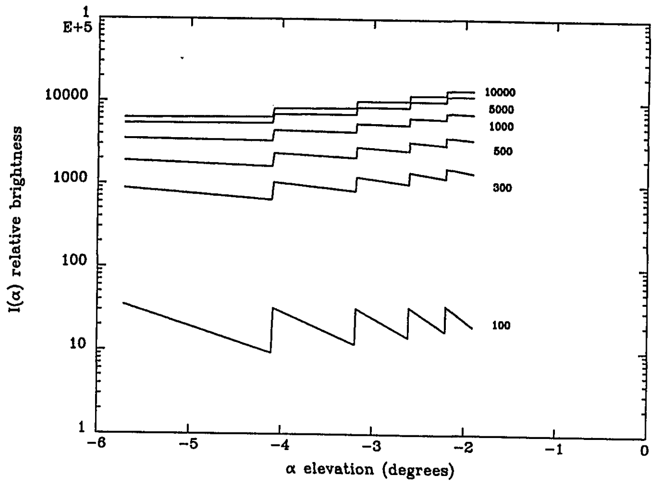

Fig. 4. Relative brightness $I(\alpha)$ versus altitude α for various scattering scale heights Ho for the geometry shown in Fig. 5.

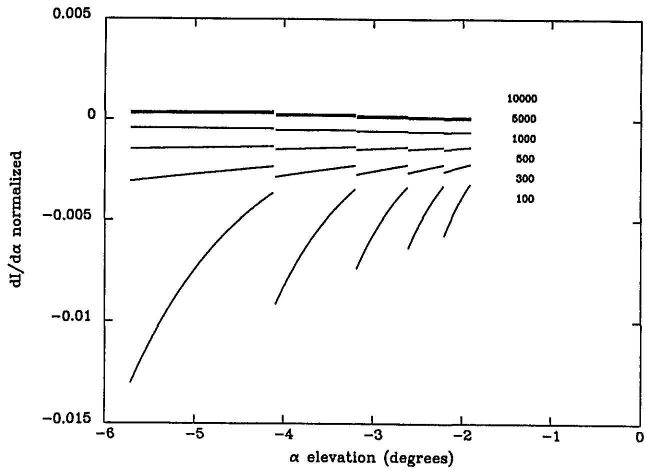

Fig. 5. Vertical brightness gradients $dI/d\alpha$ of the brightness distributions $I(\alpha)$ shown in Fig. 6.

negative for small values of Ho and positive for large Ho. The gradients are about zero for $Ho = 2000$ m, tolerably close to the value of $ho = 1000$ m as predicted by the analysis. While a factor of 2 may seem rather large to call tolerably close (the difference between the theory and numerical results probably results from the approximations made in the analysis), it must be remembered that we are attempting to show the existence of an effect, not its magnitude.

In summary, we see that in many cases $dI/d\alpha$ can be negative. Such brightness profiles are sufficiently similar to the perceived profiles that the observer's qualitative assessment that the gradient is negative is correct. In these conditions the visual illusion that results in the perception of bands, i.e., negative gradients, is not responsible for the origin of the bands, although it undoubtedly enhances the perception of bands. It is important to realize that a number of different geometric parameters were used in many simulations. In no case did the negative gradients vanish. The only thing that changed was their magnitude, i.e., the contrast of the bands, not their existence.

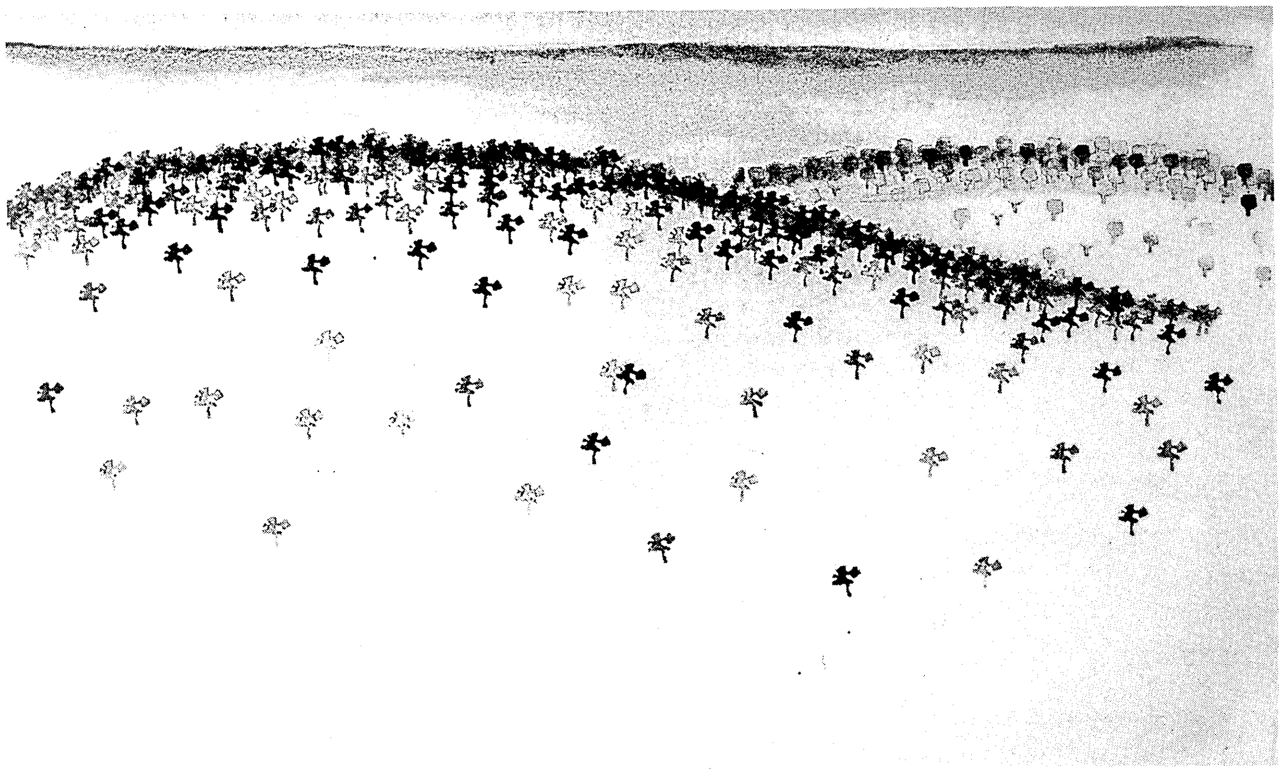

Fig. 6. Foreshortening can cause a uniform distribution of dark trees on light-colored hills to show dark bands on the ridges.

The values chosen for the final simulation (see above) were scaled directly to make Fig. 3.

Discussion

In the analysis of Minnaert's observations, I have made three assumptions, two explicit (1 and 2 below) and one implicit (3 below):

1. Observations of distant mountains exhibit the same features as observations of moors.
2. Step brightness changes are related to airlight.
3. There is nothing about the topography of a moor that would, intrinsically, cause it to show real photometric brightness variations with elevation angle.

This third assumption, which I employed by making the mountains used in the analysis black, may very well be wrong for many common situations. Minnaert makes a similar assumption by ignoring the other possible causes of brightness variations.

A series of rolling hills and especially rocky, barren, infertile hills (moors) has precisely the surface that would appear to have systematic brightness variations. For example, barren hills with light-colored soil and a sprinkling of dark rocks, or rolling grass-covered hills with a uniform distribution of trees would, from the observer's oblique point of view, have, on average, a negative brightness gradient (Fig. 6). On the other hand, it is not hard to imagine situations in which positive gradients are present. A perfectly uniform sand dune lit from above or from slightly behind will present a positive gradient as will a hill covered in dark brush with a random distribution of taller, lighter-colored bushes, or a rushy hillside backlit by the low sun.

If visual observations can lead to inaccurate assessments of brightness distributions, we might suppose

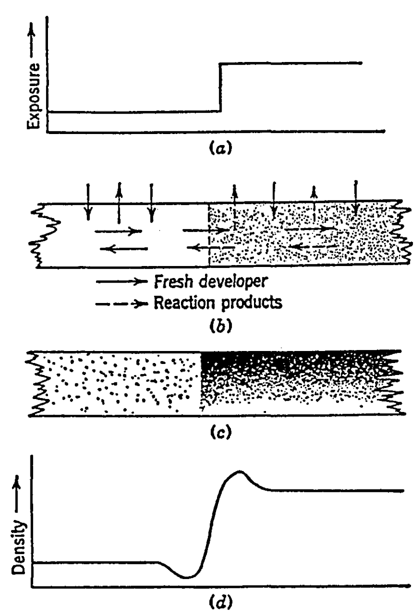

Fig. 7. Adjacency effects in photographic emulsions; reprinted from Ref. 13.

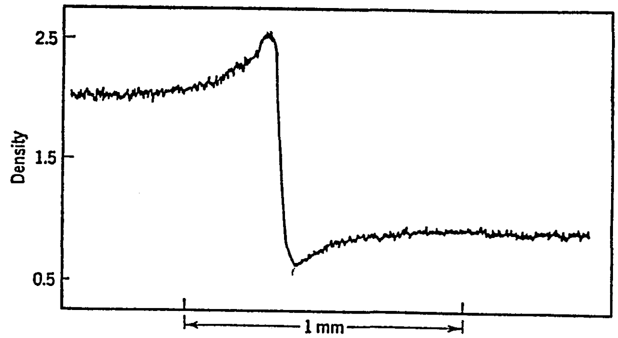

Fig. 8. Density tracing across a step brightness change photographed with conventional black-and-white film. This is an example of adjacency effects in photographic emulsions, reprinted from Ref. 13.

that photometrically calibrated photographs would be the best approach to measuring the true intensity profiles of mountains. Yet this is not true. Photographic adjacency effects[13–15] involving systematically nonuniform latent image development because of uneven depletion in chemical activity near step exposure changes actually cause densitometric variations in the image that mimic the visual illusion (Figs. 7 and 8). These nonlinear effects are most serious when there is a strong step change in density and are not expected to be as dramatic when mountain ridges are involved because of the relatively small density changes.

Conclusions

Minnaert's claim that the bands observed near the boundaries of undulating moors are visual illusions may not always be true. Common conditions may lead to bandlike structures, including airlight from scatterers with a small scale height, illumination variations, random and systematic textures viewed in perspective or seen foreshortened. Attempts to record and analyze scenes photographically can lead to inaccurate results because of nonlinear photographic effects.

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