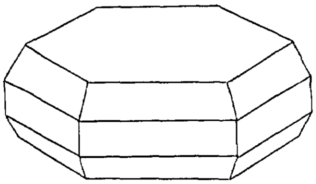

FIG. 14. Crystal with truncated pyramidal faces.

p_1p_4, respectively. The normal plane passes through the principal crystal axis. To give rise to a point of contact, the sun must lie in the normal plane and the angle of incidence of the light at the face of entry must correspond to minimum deviation. When the crystal axis is vertical, so also will be the normal plane, and the points of contact, or of nearest approach, will lie vertically above or below the sun. When the crystal axis is horizontal and the solar elevation is low, the position of the normal plane that must contain the sun for minimum deviation will lie near the horizontal. The points of contact will then lie near the 9 o'clock and 3 o'clock positions.

The two possibilities may now be compared with the photographs taken by Professor Scorer of the 1974 display (see Plate 100). Not only are they of excellent quality but they are very probably unique in providing a photographic record of halos of unusual radii. These photographs were measured very carefully by the late E. C. W. Goldie, and his results were presented by him in a joint paper with G. T. Meaden and R. White.[4] The article has incidentally a useful list of references to other papers on solar halos of unusual radii. Figure 13 shows Goldie's interpretations of Scorer's photographs. The present writer also examined the photographs soon after the event and would confirm Goldie's findings.

Scorer's photograph and Fig. 13 clearly show that associated arcs occur near halos 1 and 3 near the 12 o'clock position. Associated arcs cannot be clearly discerned in connection with any of the other halos, although there is considerable general illumination in the regions where they would occur.

These observations would appear to indicate quite unequivocally that the phenomena arose from crystals floating with their principal axes vertical. It would seem unlikely that a crystal shaped like the drawing in Fig. 1 would descend through the atmosphere in this attitude. However, the pyramidal ends to the crystal were doubtless truncated and, overall, the crystal approached more nearly the plate form than the column (Fig. 14).

This suggestion is supported by two further considerations. The first is that a weak sun pillar is clearly discernible, and the flat truncated ends would give rise to this. The second is that it is rather surprising that no very brilliant parhelion is observable. When a parhelion is produced by thin plates, the light would have to be "piped" across the crystal by means of total reflections at the top and bottom faces. The presence of the pyramidal faces would increase the distance between the top and bottom surfaces and render them less effective in getting the light across.

Although we have here such clear evidence that the crystal axes of the crystals were vertical, it must not be concluded that the other alternative, horizontal principal axes, never occurs. Crystals of the shape in Fig. 1 could well occur on other occasions and the associated arcs corresponding to this attitude of descent be produced. Only the passage of time can lead to a more definite conclusion on this point, as adequately recorded observations build up. The position of the arcs associated with the halos of Van Buijsen and Burney, with the sun low in the sky, would appear to be critical in deciding the origin of the halos in further observations.

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[2]Louis Besson, "Concerning Halos of Unusual Radii," Mon. Weather Rev., May, 254–255 (1923).
[3]R. A. R. Tricker, *Introduction to Meteorological Optics* (American-Elsevier, New York, 1970), pp. 76–78.
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Polarization models of halo phenomena. I. The parhelic circle

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An optical model of the parhelic circle is computed which includes the Stokes parameters of polarization. The model is based on one of two closely related mechanisms in crystals which are known to exist in cirrus clouds. An analysis of the circumstances of occurrence shows that one mechanism —external reflection from the side faces of ice crystal plates with c axes vertical—probably generates most parhelic circles.

INTRODUCTION

The parhelic circle is an ice halo formed in cirrus clouds.[1,2] It is colorless, passes through the sun and encircles the sky parallel to the horizon at the solar elevation. There are several mechanisms, both reflective and refractive, that could produce such a halo, though no single mechanism has yet been identified. Perhaps this is because any number of them may act together depending on the forms of ice present in the cloud. In this study we analyze the parhelic circle and argue that either of two closely related mechanisms can reproduce the observations.

I. OBSERVATIONAL CONSIDERATIONS

The parhelic circle (PHC) has several characteristics upon which a model can be constructed. First, it occurs at all azimuths relative to the sun, suggesting that an odd number of

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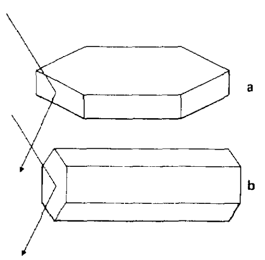

FIG. 1. Two mechanisms that form the parhelic circle. (a) External reflection from the side faces of plates oriented with their c axes vertical. (b) External reflection from the basal faces of pencil crystals. In each case the orientation about the c axis is arbitrary.

reflections is involved. An even number of reflections within the same crystal would result in an invariant deviation, perhaps as with the paranthelia. Another property of the PHC is its lack of color, again suggesting a nondispersive mechanism such as reflection, though we cannot rule out refractive dispersion parallel to the horizon, which would not be observed due to overlap of adjacent parts of the halo. The third property of the PHC that must be explained is its occurrence at the solar elevation.

II. MODEL

The simplest mechanism that can produce the parhelic circle is external reflection from vertical faces. Two known crystals have the necessary faces and they are indistinguishable from one another from the standpoint of the PHC. They are the plate with c-axis vertical (side faces reflecting) and the column with c-axis horizontal [basal face reflecting (Fig. 1)]. No further constraints on these crystals are necessary to account for any aspect of the parhelic circle. We shall investigate these mechanisms as candidates for the source of the parhelic circle.

Several other mechanisms exist that may produce the PHC. The most likely is an odd number of internal reflections from the side faces of crystals oriented as in Fig. 1(a), the light immerging and emerging through the upper and lower faces respectively. Since this would cause a considerably greater attenuation in brightness—both by reflection and by the reduced likelihood that the walls would be perfectly formed—it would create a fainter parhelic circle. For these reasons this and the other alternative mechanisms will not be discussed further.

Ice is a uniaxial negative crystal.[3] The indices of refraction are $\epsilon = 1.3104$ and $\omega = 1.309$. In the case of reflection the birefringence can be ignored and the mean value of $n = 1.31$ will be adopted as the index of refraction.

For a crystal with a unit area vertical face illuminated by a unit intensity source S (the sun), the reflected intensity is

$$1 = F(a,e) * \sin(a/2) * \cos(e), \tag{1}$$

where a and e are the azimuth and elevation of the crystal as seen by the observer O (Fig. 2). The trigonometric terms in Eq. (1) represent the reduced cross section of the crystal due to foreshortening. $F(a,e)$ is the Fresnel term

$$F(a,e) = r_p + r_s, \tag{2}$$

which is identical to the first Stokes parameter, where r_p and r_s are the reflectances

$$r_p = [\tan(i - i')]^2/[\tan(i + i')]^2, \tag{3a}$$

$$r_s = [\sin(i - i')]^2/[\sin(i + i')]^2, \tag{3b}$$

in the plane of incidence and perpendicular to the plane of incidence respectively; i and i' are the angles of incidence and refraction

$$i = \cos^{-1}[\cos(e) * \sin(a/2)], \tag{4}$$

$$i' = \sin^{-1}[(\sin(i)]/n. \tag{5}$$

The unnormalized Stokes parameters are easily defined in terms of r_p and r_s:

$$S_0 = r_p + r_s, \tag{6a}$$

$$S_1 = r_p - r_s, \tag{6b}$$

$$S_2 = 2 * (r_p * r_s)^{1/2} \cos(\phi), \tag{6c}$$

$$S_3 = 2 * (r_p * r_s)^{1/2} \sin(\phi). \tag{6d}$$

For linear polarization, the phase ϕ is 0°. The plane of incidence is defined by the sun S, the observer O and the crystal at P in the parhelic circle. In order to calculate the angle between the plane of incidence and the vertical plane, we define the vectors $\mathbf{S}$ and $\mathbf{P}$

$$\mathbf{S} = \cos(e)\hat{i} + 0\hat{j} + \sin(e)\hat{k}, \tag{7}$$

$$\mathbf{P} = \cos(e) * \cos(a)\hat{i} + \cos(e) * \sin(a)\hat{j} + \sin(e)\hat{k}, \tag{8}$$

from which θ is readily calculated as

$$\theta = 180° - \cos^{-1}\left(\frac{\sin(e)[\cos(a) - 1]}{\{\sin^2(a) + \sin^2(e) * [\cos(a) - 1]^2\}^{1/2}}\right), \tag{9}$$

where θ is measured counterclockwise from vertical if $0 < a < 180°$ and clockwise if $180° < a < 360°$.

For an observer making polarization measurements, maximum extinction occurs when the plane of the polarizing analyzer ($\mathbf{E}$ vector) is parallel to the plane of incidence, i.e., when it is aligned with θ.

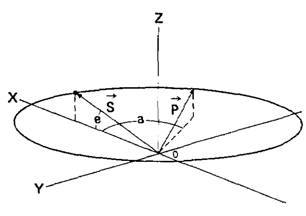

FIG. 2. Coordinate system used to analyze the parhelic circle. **S**: vector from the observer O to the sun. **P**: vector from the observer to an arbitrary point on the parhelic circle **P**. e: elevation of the sun and parhelic circle. a: azimuth of an arbitrary point on the PHC.

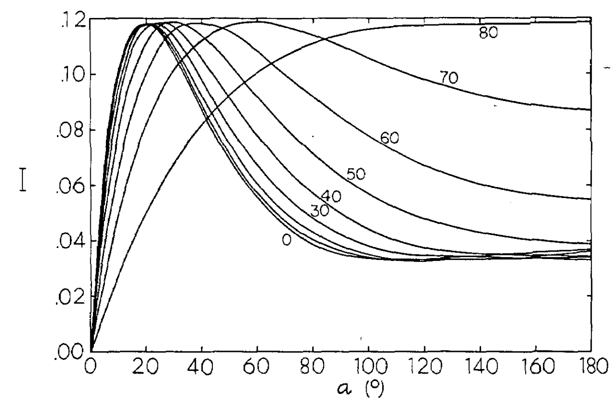

FIG. 3. Total intensity I of the PHC as a function of azimuth a, with elevation e as a parameter. Note that for low elevations (<30°) the maximum intensity occurs near 22°, in the vicinity of the parhelia.

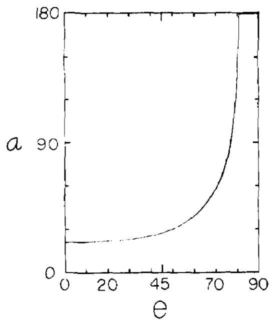

FIG. 4. Azimuth of the greatest intensity of the parhelic circle as a function of elevation.

Two other useful quantities may be calculated. The degree of polarization **D** is defined as

$$D = S_1/S_0, \tag{10}$$

which varies from 0 (unpolarized) to 1 (100% linearly polarized). The ratio r_p/r_s was also calculated because it is easily related to the observations; r_p/r_s is equal to the ratio I_p/I_s where I_p and I_s are the respective maximum and minimum intensities observed through the polarization analyzer.

III. RESULTS OF THE MODEL

The total intensity I as a function of azimuth is plotted with the elevation as a parameter in Fig. 3. For elevations 0° and 90° the intensity is identically zero because the incident ray is parallel to the reflecting surface and the cross section is reduced to zero by foreshortening. The lowest solar elevation for which the parhelic circle will occur is 0.25°—when the sun is on the horizon. This configuration represents the minimum elevation envelope for those curves in which the elevation is a parameter. Most parhelic circles are reported for low solar elevations ($e < 30°$) and it is clear from Fig. 3 that the maximum intensity is found near azimuth 22°, i.e., in the vicinity of the parhelia (sundogs) when the latter occur. Such a coincidence would complicate a photometric analysis of either halo. With increasing elevation, the azimuth of maximum intensity increases. This is shown in Fig. 4.

The degree of polarization D is shown as a function of azimuth in Fig. 5 (for convenience, D is shown). The zeroes of these functions correspond to those rays that strike the crystal at the Brewster angle 52.6° [= $\tan^{-1}(n)$], and consequently produce total polarization. The azimuth for which total polarization results is shown as a function of elevation in Fig. 6. At e = 0 the azimuth is 74.7 [= 180° − 2 $\tan^{-1}(n)$] and increases slowly with elevation. When the sun is higher than the Brewster angle (where total polarization occurs at azimuth 180°) the azimuth of maximum polarization is always 180° but the polarization is no longer total.

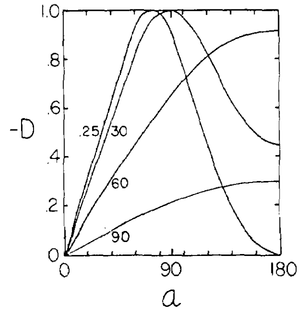

FIG. 5. Degree D of linear polarization as a function of azimuth. When the elevation is greater than 52.6°, the Brewster angle, the light from the PHC is never totally polarized.

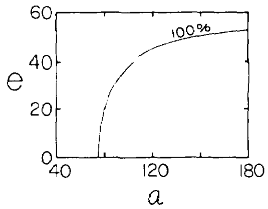

FIG. 6. Azimuth of total polarization as a function of elevation.

The angle θ that the plane of incidence makes with the vertical plane is plotted as a function of azimuth in Fig. 7. When a polarizing analyzer is aligned parallel to θ (**E** vector), maximum extinction occurs. The azimuth is found from Fig. 6. When the observer rotates the polarization analyzer, the ratio of minimum and maximum intensities can be found from the ratio r_p/r_s, which is shown in Fig. 8.

IV. DISCUSSION

This analysis has set forth the polarization properties of a parhelic circle formed by external reflection from vertical side faces of an ice crystal. Figure 1 shows two crystals that have the required faces, both of which are known to exist and produce halos. Each form produces a set of halos that are clearly distinguishable from one another. Associated with the plates [Fig. 1(a)] are the parhelia and the pillars. Pencils [Fig. 1(b)] cause the Parry arcs and upper tangent arc (circumscribed halo). In this section we shall analyze the occurrence of halos that accompany the PHC in an attempt to find out if one crystal dominates the formation of the parhelic circle.

Parhelia of the 22° halo are very common and there is little doubt that they are formed by transmission through vertical side faces of plates oriented as in Fig. 1(a).[3] Parhelia associated with the PHC may be found in Kidson,[4] Findlater,[5] and Jones and Wiggins.[6] When the incoming ray strikes the side face, part of it is reflected and must produce a parhelic circle. Indeed an analysis of the statistics of halo formation shows that most parhelic circles are attended by parhelia. However, the same analysis reveals that the PHC is far less common than the parhelia. The relative scarcity is easily understood in terms of brightness of the PHC. Even assuming that the transmitted and reflected rays are approximately equal in intensity, the PHC will be fainter than the parhelia by 10^{-2} − 10^{-3} owing to its larger angular size.

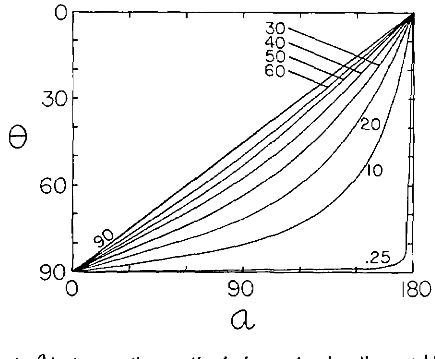

FIG. 7. Angle θ between the vertical plane at azimuth a and the plane of incidence for various elevations.

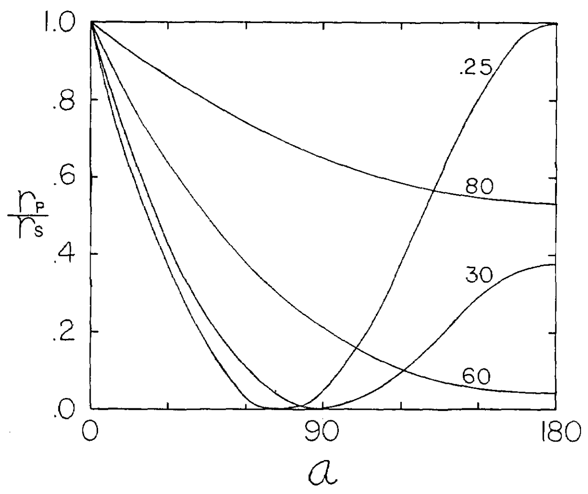

FIG. 8. Ratio of minimum-to-maximum intensity as a function of azimuth.

Simultaneous occurrences of the parhelic circle and the upper tangent arc without parhelia, columns, or other halos (which indicate the presence of plates) are infrequent presumably because the pencil crystals [Fig. 1(b)] with clean basal faces are rare, have pyramidal terminations, or are so small that diffraction smears out the PHC. Moon[7] reports a PHC with a circumscribed halo, as does Fraser.[8]

On at least one puzzling occasion (Maunsell[9]) the parhelic circle was sighted with no other halos present. Owing to the episodic occurrences of halos, the possibility of irregular cloud coverage, and the realization that we do not fully understand any halo, we mention but shall not comment further on this remarkable observation.

V. CONCLUSIONS

The main results from this study are: (i) A polarization model of the parhelic circle has been computed that makes a number of very specific predictions that can be tested observationally. (ii) Most parhelic circles are probably formed by external reflection from the side faces of plates whose c axis is vertical. These are the same crystals that produce the common parhelia. (iii) The most sensitive test of the model could be conducted on parhelic circles occurring at low (<30°) elevations and would consist of maximum-to-minimum intensity ratio measurements by a polarizing analyzer as a function of azimuth.

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[6]G. A. Jones and K. J. Wiggins, "Halo Phenomena at Odiham," Weather **19**, 289–290 (1964).
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[8]A. B. Fraser (private communication at OSA meeting Keystone, Colorado), 1978.
[9]F. G. Maunsell, "Parhelic Circle," Weather **6**, 245 (1951).

Arcs of Lowitz

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A computer simulation technique is used to investigate the origins of the arcs of Lowitz. The model explored consists of light passing through a hexagonal ice plate, spinning about a major diagonal axis that remains horizontal as the crystal falls.

INTRODUCTION

Many halos, arcs, and streaks of light in the sky result from the refraction of sunlight by falling ice crystals. The arcs of Lowitz are a comparatively rare effect but one whose reality seems fairly certain. First noted by Tobias Lowitz in the famous St. Petersburg halo display of 1790,[1] the arcs are usually described as being tangent to the 22° halo below the parhelic circle and extending upward to the parhelia. Subsequent sightings of the Lowitz arcs are not numerous, but some reports do exist in the literature.[2]

The suggested explanation for the arcs of Lowitz involves light rays passing through alternate faces of right hexagonal ice prisms in the shape of flat plates, with basal faces much larger than the side faces. Most of the theories of these arcs involve crystals that spin about a horizontal axis as they fall—this horizontal axis being parallel to the basal plane of the crystal. This spinning mode can be demonstrated easily with a computer card if you hold the card horizontally by grasping it between thumb and finger in the middle of a long edge. When you release it from this position it will spin about its long axis as it falls. The motion can be explained with simple physical arguments.[3]

A feature of this spinning motion, applied to a flat plate

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