# Intensity profile of the $22^{\circ}$ halo 

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We report the first relative intensity measurements made to our knowledge of the $22^{\circ}$ halo, obtained from photographic photometry of a halo of exceptional brightness and uniformity. The maximum brightness occurs at $22.8^{\circ}$, and a relative minimum occurs at $19.7^{\circ}$. The full width at one half the maximum intensity is $3.4^{\circ}$. A low-intensity tail reaches from $29^{\circ}$ to $39^{\circ}$.

## INTRODUCTION

Despite the growing scientific awareness of ice-crystal halos, there is surprising lack of quantitative observational data about them. Even the most basic (and often-referenced) angular measurements sometimes consist of casual determinations of the radii of circular halos with no mention whatever of the intensity distribution. More-complex halos are reported with either freehand drawings or photographs. Seldom if ever are the photographs measured to ascertain the trajectories of noncircular halos. Intensity measurements are unknown, the closest being a smoothed, uncalibrated photographic density scan across a $22^{\circ}$ halo by Bruche and Bruche. ${ }^{1}$ Quantitative color measurements do not exist for any halo, and only recently have polarization studies been made by Lynch ${ }^{2}$ and Konnen. ${ }^{3}$ Most of the growing wealth of information about ice crystal halos is theoretical because naturally occurring halos are short lived, episodic, and seldom occur under the conditions necessary for detailed observation. Still, the most basic properties of halos (intensity versus angular position on the sky) are not quantitatively known.

In this paper we present quantitative measurements of the intensity distribution of the $22^{\circ}$ halo.

## OBSERVATIONS OF THE $22^{\circ}$ HALO

The halo observations reported here were made in Malibu, California, on the evening of December 3, 1979. An exceptionally bright and well-defined $22^{\circ}$ halo was observed by moonlight (Fig. 1). Its contrast was very high, and the halo was almost perfectly uniform around its circumference. The cirrostratus clouds showed little texture or anisotropy and covered the sky so evenly that their presence could only be inferred from the halo. The moon was approximately $50^{\circ}$ above the horizon. A number of photographs were made on Kodak Plus-X pan film with exposure times ranging between 1 and 10 sec . During this time the moon would have moved no more than $0.04^{\circ}$, an insignificant amount compared with the diameter of the moon $\left(0.5^{\circ}\right)$ and with the width of the halos $\left(\simeq 3^{\circ}\right)$. The images of the moon and the halo were carefully centered in the frame to render any image distortion symmetric on the film and to prevent lens ghosts from falling on the halo proper. The following day several exposures of calibrated intensity wedges and grids were made with the same equipment, thereby calibrating intensity on the film and the angular scale and distortion of the $16-\mathrm{mm}$ lens. A scien-
tific laboratory developed the film according to the manufacturer's instructions.

The images of the halo and calibration wedge were digitized using a spot size of $25 \mu \mathrm{~m}\left(0.18^{\circ}\right)$. After the photographic density was converted to true relative intensity (Figs. 2A and 2B), the computerized image was measured to locate the moon, the assumed center of the halo. The location of each pixel (deviation angle $D$ relative to the moon) was calculated, and the entire upper half of the two-dimensional image (approximately 113,000 data points) was reduced to a plot of relative intensity versus deviation angle (angle on the sky) along with an average curve calculated by summing intensity pixels in $0.1^{\circ}$ window intervals (Fig. 2C; Table 1, column 1). In reducing the data to a plot of brightness versus angle we assumed that there was no azimuthal variation in brightness based on the comparison of several radial intensity profiles. The lower half of the halo was not used owing to concerns about absorption by the large air mass at large zenith distances.

If there were no halo present, scattering by air molecules and cloud particles would result in a brightness distribution that decreased monotonically away from the moon. In order to remove this background trend we assumed that the halo intensity distribution was independent of the background and represented an additive component to the sky brightness. Since the halo is not expected to contribute light at angles outside its minimum and maximum deviation angles ( $21.8^{\circ}$ and $43.5^{\circ}$, respectively, for no internal reflections in the crystal), we fitted a third-order polynomial to these parts of the intensity distribution and subtracted it from the curve (Fig. 2C). The fit was extended from $15^{\circ}$ (the maximum extent of lens flare on the image) to $45^{\circ}$, the maximum sky angle on the film excluding the halo regions between $21.8^{\circ}$ and $43.5^{\circ}$. The result is shown in Fig. 2D and should represent the residual halo scattering intensity as a function of deviation angle $D$ measured from the light source. For ease of comparison with this and other theories and observations, Table 1 lists the raw and subtracted normalized intensity distribution for the $22^{\circ}$ halo shown in Fig. 1. The effect of misjudging the position of the true background is to shift the curve (Fig. 2D; Table 1, column 3) up or down by a small constant amount and has the greatest effect far out in the profile $\left(>28^{\circ}\right)$. The effect on the values near minimum deviation ( $\simeq 22^{\circ}$ ) is small.

The observed intensity profile (Fig. 2; Table 1, column 2) shows a minimum centered at $19.7^{\circ}$. Both observed and


Fig. 1. $22^{\circ}$ halo observed on December 3, 1979, in Malibu, California, by the author. The halo was remarkable for its brightness, high contrast, and azimuthal uniformity and because the lunar altitude was about $50^{\circ}$, well above the radius of the halo. No other halos were observed during the display, and the cirrus clouds were uniform and showed virtually no structure or anisotropy.
corrected profiles show a halo brightness maximum at $22.8^{\circ}$, in good agreement with simple minimum-deviation calculations and more-sophisticated theoretical models. The full width at one half of the maximum intensity (FWHM) is $3.4^{\circ}$. A broad tail occurs between about $29^{\circ}$ and $39^{\circ}$.

## DISCUSSION

The simplest models of the $22^{\circ}$ halo that include the finite width of the sun and the dispersion of light in ice predict a FWHM of the radial profile of $2.5^{\circ},{ }^{4}$ considerably narrower than our observed profile. If the discrepancy is due to diffraction by the ice crystals (an effect not included in model calculations), we can estimate the size of the crystal responsible for this halo. The diffraction pattern for a slit of width $a$ is given by

$$
\left(I / I_{0}\right)=(\sin \alpha / \alpha)^{2}
$$

where

$$
\alpha=(\pi a / \lambda) \sin (\theta)
$$

where $\theta$ is the diffraction angle, $\lambda$ is the wavelength, and $a$ is the diffraction aperture width. The half-power widths (FWHM) occur when $\alpha=1.40$. Using $\lambda=0.5 \mu \mathrm{~m}$ and $\sin \theta$ $=\theta$, we find a simple expression for the width $W_{\text {dif }}$ of the diffraction pattern's first maximum of

$$
W_{\mathrm{dif}}=25.5 / a
$$

where $W_{\text {dif }}$ is measured in degrees. Since the observed intensity profile is not highly asymmetric, we may estimate the diffraction width $W_{\text {dif }}$ of the halo by deconvolving the theoretical width $W_{\text {th }}$ from the observed width $W_{\text {obs }}$ using the quadrature rule for deconvoluting Gaussians, i.e.,

$$
\left(W_{\text {obs }}\right)^{2}=\left(W_{\mathrm{th}}\right)^{2}+\left(W_{\mathrm{dif}}\right)^{2},
$$

resulting in $W_{\text {dif }} \simeq 2.5^{\circ}$ and $a \simeq 10 \mu \mathrm{~m}$. This is only a rough estimate ( $\pm 30 \%$ ) because projection effects, index-of-refraction variations, etc. have not been included. The value of $a$ is clearly in the size range for crystals that are expected to be randomly oriented (Nikiforova et al. ${ }^{5}$ )

Since no $46^{\circ}$ halo was observed during the December 3, 1979, display we might suppose that the aspect ratio of the crystal was much greater than 1 , i.e., the cirrus clouds were composed primarily of columns rather than of plates (Lynch, ${ }^{4}$ Pattlock and Trankle ${ }^{6}$ ). It is also possible that any hexagonal ice crystal without basal pinacoids could cause the $22^{\circ}$ halo but not the $46^{\circ}$ halo. The most likely candidate are pyramidally terminated bullet crystals.
The profile information reported here is for only one halo, a halo that was not necessarily characteristic of all halos. Indeed, its exceptional brightness may have been due not simply to a larger-than-usual number of scattering crystals


Fig. 2. Reduction of raw data to intensity profile of the $22^{\circ}$ halo. A, Photographic density profile for a single vertical scan through the halo and the moon. B, Same scan as A after conversion to relative intensity. C, Averaged intensity profile using approximately 113,000 points. D, Averaged intensity profile after removal of the background trend indicated in C .
but rather to a distribution of crystals (or other effects) that not only rendered the halo bright but also caused its intensity distribution to be different from that of other $22^{\circ}$ halos. Although we do not believe that this is likely, only further photometric measurements of other halos can settle the question.

## SUMMMARY AND CONCLUSIONS

We have made the first reported photometric measurements of an ice-crystal halo and compared the radial intensity profile with a first-order theoretical model for light refracted and diffracted through the crystal in the plane perpendicular to the refracting edge. The difference between observed and theoretical full widths of the halos suggested that diffraction by crystals approximately $10 \mu \mathrm{~m}$ across their refracting faces widens the halo. Since the $46^{\circ}$ halo was not observed, the crystals responsible for the halo reported here were probably

Table 1. Photometric Measurements of the Radial Halo Profile

| $D\left({ }^{\circ}{ }^{a}\right.$ | $I_{\text {rel }}{ }^{\text {b }}$ | $I_{\text {halo }}{ }^{\text {c }}$ |
| :---: | :---: | :---: |
| 15.0 | 4.80 | 0.000 |
| 15.1 | 3.15 | 0.000 |
| 15.2 | 3.17 | 0.000 |
| 15.3 | 3.07 | 0.000 |
| 15.4 | 3.03 | 0.000 |
| 15.5 | 3.04 | 0.000 |
| 15.6 | 3.00 | 0.000 |
| 15.7 | 2.96 | 0.000 |
| 15.8 | 2.89 | 0.000 |
| 15.9 | 2.87 | 0.000 |
| 16.0 | 2.86 | 0.000 |
| 16.1 | 2.82 | 0.000 |
| 16.2 | 2.78 | 0.000 |
| 16.3 | 2.72 | 0.000 |
| 16.4 | 2.75 | 0.000 |
| 16.5 | 2.71 | 0.000 |
| 16.6 | 2.70 | 0.000 |
| 16.7 | 2.63 | 0.000 |
| 16.8 | 2.57 | 0.000 |
| 16.9 | 2.59 | 0.000 |
| 17.0 | 2.59 | 0.000 |
| 17.1 | 2.52 | 0.000 |
| 17.2 | 2.57 | 0.000 |
| 17.3 | 2.51 | 0.000 |
| 17.4 | 2.51 | 0.000 |
| 17.5 | 2.50 | 0.000 |
| 17.6 | 2.49 | 0.000 |
| 17.7 | 2.51 | 0.000 |
| 17.8 | 2.44 | 0.000 |
| 17.9 | 2.50 | 0.000 |
| 18.0 | 2.46 | 0.000 |
| 18.1 | 2.44 | 0.000 |
| 18.2 | 2.44 | 0.000 |
| 18.3 | 2.42 | 0.000 |
| 18.4 | 2.46 | 0.000 |
| 18.5 | 2.42 | 0.000 |
| 18.6 | 2.39 | 0.000 |
| 18.7 | 2.39 | 0.000 |
| 18.9 | 2.38 | 0.000 |
| 19.0 | 2.38 | 0.022 |
| 19.1 | 2.39 | 0.041 |
| 19.2 | 2.39 | 0.040 |
| 19.3 | 2.40 | 0.056 |
| 19.4 | 2.39 | 0.052 |
| 19.5 | 2.39 | 0.059 |
| 19.6 | 2.40 | 0.071 |
| 19.7 | 2.37 | 0.051 |
| 19.8 | 2.38 | 0.068 |
| 19.9 | 2.38 | 0.076 |
| 20.0 | 2.40 | 0.094 |
| 20.1 | 2.40 | 0.105 |
| 20.2 | 2.38 | 0.094 |
| 20.3 | 2.39 | 0.110 |
| 20.4 | 2.40 | 0.123 |
| 20.5 | 2.42 | 0.142 |
| 20.6 | 2.41 | 0.139 |
| 20.7 | 2.43 | 0.166 |
| 20.8 | 2.48 | 0.218 |
| 20.9 | 2.53 | 0.268 |
| 21.0 | 2.52 | 0.264 |
| 21.1 | 2.52 | 0.273 |
| 21.2 | 2.57 | 0.322 |
| 21.3 | 2.65 | 0.400 |

Table 1. Continued


Table 1. Continued

| $D\left({ }^{\circ}\right)^{a}$ | $I_{\text {rel }}{ }^{\text {b }}$ | $I_{\text {halo }}{ }^{\text {c }}$ | $D\left({ }^{\circ}\right)^{a}$ | $I_{\text {rel }}{ }^{\text {b }}$ | $I_{\text {halo }}{ }^{\text {c }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 34.2 | 1.80 | 0.047 | 39.7 | 1.63 | 0.000 |
| 34.3 | 1.78 | 0.036 | 39.8 | 1.63 | 0.000 |
| 34.4 | 1.79 | 0.042 | 39.9 | 1.63 | 0.000 |
| 34.5 | 1.79 | 0.050 | 40.0 | 1.62 | 0.000 |
| 34.6 | 1.79 | 0.048 | 40.1 | 1.64 | 0.000 |
| 34.7 | 1.78 | 0.044 | 40.2 | 1.63 | 0.000 |
| 34.8 | 1.77 | 0.040 | 40.3 | 1.62 | 0.000 |
| 34.9 | 1.77 | 0.044 | 40.4 | 1.62 | 0.000 |
| 35.0 | 1.77 | 0.039 | 40.5 | 1.62 | 0.000 |
| 35.1 | 1.78 | 0.053 | 40.6 | 1.63 | 0.000 |
| 35.2 | 1.76 | 0.032 | 40.7 | 1.63 | 0.000 |
| 35.3 | 1.75 | 0.030 | 40.8 | 1.62 | 0.000 |
| 35.4 | 1.75 | 0.035 | 40.9 | 1.63 | 0.000 |
| 35.5 | 1.76 | 0.041 | 41.0 | 1.61 | 0.000 |
| 35.6 | 1.75 | 0.035 | 41.1 | 1.62 | 0.000 |
| 35.7 | 1.74 | 0.029 | 41.2 | 1.62 | 0.000 |
| 35.8 | 1.75 | 0.038 | 41.3 | 1.63 | 0.000 |
| 35.9 | 1.74 | 0.031 | 41.4 | 1.63 | 0.000 |
| 36.0 | 1.73 | 0.020 | 41.5 | 1.63 | 0.000 |
| 36.1 | 1.73 | 0.024 | 41.6 | 1.65 | 0.000 |
| 36.2 | 1.73 | 0.025 | 41.7 | 1.62 | 0.000 |
| 36.3 | 1.72 | 0.021 | 41.8 | 1.64 | 0.000 |
| 36.4 | 1.71 | 0.017 | 41.9 | 1.62 | 0.000 |
| 36.5 | 1.73 | 0.036 | 42.0 | 1.63 | 0.000 |
| 36.6 | 1.72 | 0.024 | 42.1 | 1.63 | 0.000 |
| 36.7 | 1.71 | 0.014 | 42.2 | 1.64 | 0.000 |
| 36.8 | 1.70 | 0.010 | 42.3 | 1.65 | 0.000 |
| 36.9 | 1.70 | 0.015 | 42.4 | 1.64 | 0.000 |
| 37.0 | 1.70 | 0.009 | 42.5 | 1.64 | 0.000 |
| 37.1 | 1.71 | 0.022 | 42.6 | 1.63 | 0.000 |
| 37.2 | 1.70 | 0.015 | 42.7 | 1.65 | 0.000 |
| 37.3 | 1.70 | 0.016 | 42.8 | 1.65 | 0.000 |
| 37.4 | 1.69 | 0.015 | 42.9 | 1.65 | 0.000 |
| 37.5 | 1.69 | 0.012 | 43.0 | 1.65 | 0.000 |
| 37.6 | 1.69 | 0.015 | 43.1 | 1.64 | 0.000 |
| 37.7 | 1.68 | 0.005 | 43.2 | 1.68 | 0.000 |
| 37.8 | 1.69 | 0.019 | 43.3 | 1.66 | 0.000 |
| 37.9 | 1.68 | 0.011 | 43.4 | 1.65 | 0.000 |
| 38.0 | 1.66 | 0.000 | 43.5 | 1.65 | 0.000 |
| 38.1 | 1.67 | 0.009 | 43.6 | 1.64 | 0.000 |
| 38.2 | 1.67 | 0.010 | 43.7 | 1.63 | 0.000 |
| 38.3 | 1.67 | 0.010 | 43.8 | 1.63 | 0.000 |
| 38.4 | 1.66 | 0.000 | 43.9 | 1.64 | 0.000 |
| 38.5 | 1.66 | 0.007 | 44.0 | 1.63 | 0.000 |
| 38.6 | 1.67 | 0.017 | 44.1 | 1.63 | 0.000 |
| 38.7 | 1.66 | 0.007 | 44.2 | 1.63 | 0.000 |
| 38.8 | 1.65 | 0.004 | 44.3 | 1.63 | 0.000 |
| 38.9 | 1.65 | 0.001 | 44.4 | 1.63 | 0.000 |
| 39.0 | 1.65 | 0.000 | 44.5 | 1.63 | 0.000 |
| 39.1 | 1.64 | 0.000 | 44.6 | 1.63 | 0.000 |
| 39.2 | 1.64 | 0.000 | 44.7 | 1.62 | 0.000 |
| 39.3 | 1.63 | 0.000 | 44.8 | 1.62 | 0.000 |
| 39.4 | 1.65 | 0.000 | 44.9 | 1.62 | 0.000 |
| 39.5 | 1.63 | 0.000 | 45.0 | 1.61 | 0.000 |
| 39.6 | 1.64 | 0.000 |  |  |  |

${ }^{a}$ Deviation angle $D$ on sky measured from the moon.
${ }^{b}$ Relative intensity of the sky brightness before removal of background (Fig. 2C).
${ }^{c}$ Normalized intensity of profile of the $22^{\circ}$ halo with the background removed (Fig. 2D).
randomly oriented columns or pyramidally terminated crystals.

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